

1.  $\int \frac{x^2-1}{x^4+3x^2+1} dx$  is equal to [1]  
a)  $\tan\left(x + \frac{1}{x}\right) + C$  b)  $\tan^{-1}\left(x + \frac{1}{x}\right) + C$   
c)  $\tan^{-1}(3x^2 + 2x) + C$  d)  $\tan^{-1}(x^2 + 1) + C$
2. The angle between a line with direction ratios 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12) [1]  
a)  $\cos^{-1}\left(\frac{2}{3}\right)$  b)  $\tan^{-1}\left(-\frac{2}{3}\right)$   
c) none of these d)  $\cos^{-1}\left(\frac{3}{2}\right)$
3. If a unit vector  $\vec{a}$  makes angles  $\frac{\pi}{3}$  with  $\hat{i}$ ,  $\frac{\pi}{4}$  with  $\hat{j}$  and an acute angle  $\theta$  with  $\hat{k}$ , then find  $\theta$  [1]  
a)  $2\frac{\pi}{3}$  b)  $\frac{\pi}{10}$   
c)  $\frac{\pi}{3}$  d)  $\frac{\pi}{5}$
4. If one ball is drawn at random from each of three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, then the probability that 2 white and 1 black balls will be drawn is [1]  
a)  $\frac{1}{4}$  b)  $\frac{13}{32}$   
c)  $\frac{1}{32}$  d)  $\frac{3}{16}$
5.  $\int 2x^3 e^{x^2} dx = ?$  [1]  
a)  $e^{x^2}(x^2 - 1) + C$  b)  $e^{x^2}(x^2 + 1) + C$

c)  $e^{x^2} (x + 1) + C$

d) None of these

6. The probability that an event E occurs in one trial is 0.4. Three independent trials of the experiment are performed. What is the probability that E occurs at least once? [1]

a) 0.936

b) 0.784

c) None of these

d) 0.964

7. The area of the region (in square units) bounded by the curve  $x^2 = 4y$ , line  $x = 2$  and x-axis is [1]

a)  $\frac{8}{3}$

b) 1

c)  $\frac{2}{3}$

d)  $\frac{4}{3}$

8. The lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{5}$  are [1]

a) none of these

b) parallel

c) intersecting

d) skew

9. If  $\theta$  is the angle between two vectors  $\vec{a}$  and  $\vec{b}$ , then  $\vec{a} \cdot \vec{b} \geq 0$  only when [1]

a)  $0 < \theta < \frac{\pi}{2}$

b)  $0 \leq \theta \leq \pi$

c)  $0 < \theta < \pi$

d)  $0 \leq \theta \leq \frac{\pi}{2}$

10. Which of the following is a homogeneous differential equation? [1]

a)  $y^2 dx + (x^2 - xy - y^2) dy = 0$

b)  $(xy) dx - (x^3 + y^3) dy = 0$

c)  $(x^3 + 2y^2) dx + 2xy dy = 0$

d)  $(4x + 6y + 5) dy - (3y + 2x + 4) dx = 0$

11. The area bounded by the lines  $y = 2 + x$ ,  $y = 2 - x$  and  $x = 2$  is [1]

a) 16

b) 8

c) 3

d) 4

12.  $\int \frac{x^2}{(a^6 - x^6)} dx = ?$  [1]

a)  $\frac{1}{3a^3} \log \left| \frac{a^3 + x^3}{a^3 - x^3} \right| + C$

b)  $\frac{1}{6a^3} \log \left| \frac{a^3 - x^3}{a^3 + x^3} \right| + C$

c)  $\frac{1}{6a^3} \log \left| \frac{a^3 + x^3}{a^3 - x^3} \right| + C$

d) None of these

13. If the function  $f(x) = 2 \tan x + (2a + 1) \log_e |\sec x| + (a - 2)x$  is increasing on  $R$ , then [1]

$a \in R$

a)  $a = \frac{1}{2}$

b)

c)  $a \in (\frac{1}{2}, \infty)$

d)  $a \in (-\frac{1}{2}, \frac{1}{2})$

14. If  $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , then  $A^4 =$  [1]

a)  $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

b)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c)  $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

d)  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

15. If  $A$  is a matrix of order 3 and  $|A| = 8$ , then  $|\text{adj } A| =$  [1]

a) 2

b) 1

c)  $2^6$

d)  $2^3$

16. The system of equations [1]  
 $x + y + z = 2,$   
 $3x - y + 2z = 6$   
 $3x + y + z = -18$   
 has:

a) zero solution as the only solution

b) an infinite number of solutions

c) a unique solution

d) no solution

17. Range of  $\cos^{-1}x$  is [1]

a)  $[\frac{-\pi}{2}, \frac{\pi}{2}]$

b)  $[\frac{-\pi}{2}, \frac{\pi}{2}] - \{0\}$

c) None of these

d)  $(\frac{-\pi}{2}, \frac{\pi}{2})$

18. Find a solution of  $\cos\left(\frac{dy}{dx}\right) = a$  ( $a \in R$ ) which satisfy the condition  $y = 1$  when  $x = 0$ . [1]

a)  $\cos \frac{y-10}{x} = a$

b)  $\cos \frac{y-1}{x} = a$

c)  $\cos \frac{y-4}{x} = a$

d)  $\cos \frac{y-3}{x} = a$

19. **Assertion (A):** The function  $f(x) = x^2 - 4x + 6$  is strictly increasing in the interval  $(2, \infty)$ . [1]

**Reason (R):** The function  $f(x) = x^2 - 4x + 6$  is strictly decreasing in the interval  $(-\infty, 2)$ .

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If  $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$  then  $x = \pm 6$ . [1]

**Reason (R):** If A is a skew-symmetric matrix of odd order, then  $|A| = 0$ .

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

### Section B

21.  $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$  [2]

22. Solve the differential equation:  $x \frac{dy}{dx} = x + y$  [2]

23. For what value of x, the matrix  $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$  is singular? [2]

OR

Solve the system of equations by Cramer's rule:

$$2x - y = 17$$

$$3x + 5y = 6$$

24. If  $\vec{a} = (2\hat{i} - 4\hat{j} + 5\hat{k})$  then find the value of  $\lambda$  so that  $\lambda\vec{a}$  may be a unit vector. [2]

25. A bag contains 5 white, 7 red and 4 black balls. If four balls are drawn one by one with replacement, what is the probability that none is white? [2]

### Section C

26. If  $f\left(\frac{3x-4}{3x+4}\right) = x + 2$ , then find value of  $\int f(x)dx$  [3]

27. Solve the differential equation:  $\cos x \frac{dy}{dx} - \cos 2x = \cos 3x$  [3]

OR

Solve the differential equation:  $(2x^2y + y^3)dx + (xy^2 - 3x^3)dy = 0$

28. Find  $\lambda$  when the projection of  $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$  on  $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$  is 4 units. [3]

OR

Show that the points A, B, C with position vectors  $(3\hat{i} - 2\hat{j} + 4\hat{k})$ ,  $(\hat{i} + \hat{j} + \hat{k})$  and  $(-\hat{i} + 4\hat{j} - 2\hat{k})$  respectively are collinear.

29. Evaluate the integral:  $\int (x+1)\sqrt{x^2-x+1} dx$  [3]

OR

Evaluate  $\int_0^\pi \frac{1}{3+2\sin x + \cos x} dx$

30. Find  $\frac{dy}{dx}$ , when  $x = ae^\theta(\sin \theta - \cos \theta)$ ,  $y = ae^\theta(\sin \theta + \cos \theta)$  [3]

31. Find the area of the segment of the parabola  $y = x^2 - 5x + 15$  cut off by the straight line  $y = 3x + 3$ . [3]

### Section D

32. Solve the following LPP graphically: [5]

Maximize  $Z = 5x + 7y$

Subject to

$x + y \leq 4$

$3x + 8y \leq 24$

$10x + 7y \leq 35$

$x, y \geq 0$

33. Let  $R$  be a relation on  $N \times N$ , defined by  $(a, b) R (c, d) \Leftrightarrow a + d = b + c$  for all  $(a, b), (c, d) \in N \times N$ . Show that  $R$  is an equivalence relation. [5]

OR

Let  $n$  be a fixed positive integer. Define a relation  $R$  in  $Z$  as follows  $\forall a, b \in Z$   $aRb$  if and only if  $a-b$  is divisible by  $n$ . Show that  $R$  is an equivalence relation.

34. Find the shortest between the  $l_1$  and  $l_2$  whose vector equations are [5]

$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$

and  $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

OR

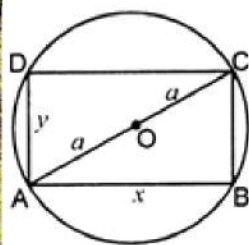
Find the distance of the point  $(-1, -5, -10)$  from the point of intersection of the line  $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$  and the plane  $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ .

35. Differentiate  $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$  with respect to  $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ , if  $0 < x < 1$ . [5]

### Section E

36. Read the text carefully and answer the questions: [4]

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)



- Find the perimeter of rectangle in terms of any one side and radius of circle.
- Find critical points to maximize the perimeter of rectangle?
- Check for maximum or minimum value of perimeter at critical point.

OR

If a rectangle of the maximum perimeter which can be inscribed in a circle of radius 10 cm is square, then the perimeter of region.

37. **Read the text carefully and answer the questions:**

[4]

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats, and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold by school A, B, C are given below.



| Article | School | A  | B  | C  |
|---------|--------|----|----|----|
| Fans    |        | 40 | 25 | 35 |
| Mats    |        | 50 | 40 | 50 |
| Plates  |        | 20 | 30 | 40 |

- Represent the sale of handmade fans, mats and plates by three schools A, B and C and the sale prices (in ₹) of given products per unit, in matrix form.
- Find the funds collected by school A, B and C by selling the given articles.
- If they increase the cost price of each unit by 20%, then write the matrix representing new price.

OR

Find the total funds collected for the required purpose after 20% hike in price.

38. **Read the text carefully and answer the questions:**

[4]

In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03.



- The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form

selected at random has an error, find the probability that the form is NOT processed by Govind.

- (ii) Find the probability that Priyanka processed the form and committed an error.

## SOLUTION

### Section A

1. (b)  $\tan^{-1}\left(x + \frac{1}{x}\right) + C$

**Explanation:** Divide num. and deno. by  $x^2$

Substitute  $x + \frac{1}{x} = t$  then  $(1 - \frac{1}{x^2})dx = dt$

$$\Rightarrow \int \frac{dt}{t^2 + 1}$$

$$\Rightarrow \tan^{-1}\left(x + \frac{1}{x}\right) + C$$

2. (a)  $\cos^{-1}\left(\frac{2}{3}\right)$

**Explanation:** The angle between a line with direction ratios 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12)

Direction ratios of the line joining the points A(3, 1, 4), B(7, 2, 12) is  $\langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle = \langle 7 - 3, 2 - 1, 12 - 4 \rangle = \langle 4, 1, 8 \rangle$

Now as the angle between two lines having direction ratios  $\langle a_1, b_1, c_1 \rangle$  and  $\langle a_2, b_2, c_2 \rangle$  is given by

$$\cos^{-1} \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Using the values we have

$$\cos^{-1} \frac{2 \times 4 + 2 \times 1 + 1 \times 8}{\sqrt{2^2 + 2^2 + 1^2} \sqrt{4^2 + 1^2 + 8^2}} = \cos^{-1} \frac{18}{27} = \cos^{-1} \frac{2}{3}$$

3. (c)  $\frac{\pi}{3}$

**Explanation:** Let

$$\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k},$$

It is given that  $|\vec{a}| = 1$

, then,

$$\Rightarrow a_1^2 + a_2^2 + a_3^2 = 1 \dots (1)$$

$$\therefore \vec{a} \cdot \hat{i} = (a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}) \cdot \hat{i}$$

$$\Rightarrow |\vec{a}| |\hat{i}| \cos \frac{\pi}{3} = a_1 \Rightarrow a_1 = \frac{1}{2}$$

$$\therefore \vec{a} \cdot \hat{j} = (a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}) \cdot \hat{j}$$

$$\Rightarrow |\vec{a}| |\hat{j}| \cos \frac{\pi}{4} = a_2 \Rightarrow a_2 = \frac{1}{\sqrt{2}}$$

$$\therefore \vec{a} \cdot \hat{k} = (a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}) \cdot \hat{k}$$

$$\Rightarrow |\vec{a}| |\hat{k}| \cos \theta = a_3 \Rightarrow a_3 = \cos \theta$$

Putting these values in (1), we get:

$$\frac{1}{4} + \frac{1}{2} + \cos^2 \theta = 1$$

$$\Rightarrow \frac{3}{4} = 1 - \cos^2 \theta \Rightarrow \sin^2 \theta = \frac{3}{4} \Rightarrow \sin \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60^\circ$$

4. (b)  $\frac{13}{32}$

**Explanation:** Here, the three boxes contain 3 white and 1 black (3 W, 1 B), 2 white and 2 black (2 W, 2 B) and 1 white and 3 black balls (1 W, 3 B), respectively

$$\begin{aligned} P(2 \text{ W}, 1 \text{ B}) &= \frac{3}{4} \times \frac{2}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4} \\ &= \frac{18}{64} + \frac{6}{64} + \frac{2}{64} \\ &= \frac{26}{64} \\ &= \frac{13}{32} \end{aligned}$$

5. (a)  $e^{x^2} (x^2 - 1) + C$

**Explanation:**  $I = \int (2x)x^2 e^{x^2} dx = \int t I e^t I dt$ , where  $x^2 = t$   
after solving we get

$$I = e^{x^2} (x^2 - 1) + C$$

6. (b) 0.784

**Explanation:**  $p = 0.4$ ,  $q = (1 - 0.4) = 0.6$  and  $n = 3$ .

Required probability =  $P(\text{E occurring at least once})$

$$\begin{aligned} &= {}^3C_1 (0.4)^1 \times (0.6)^2 + {}^3C_2 (0.4)^2 \times (0.6)^1 + {}^3C_3 (0.4)^3 \\ &= \left( \frac{54}{125} + \frac{36}{125} + \frac{8}{125} \right) \\ &= \frac{98}{125} \\ &= 0.784. \end{aligned}$$

7. (c)  $\frac{2}{3}$

**Explanation:** The area of the region bounded by the curve  $x^2 = 4y$  and line  $x = 2$  and x-axis

$$\Rightarrow \int_0^2 y dx = \int_0^2 \frac{x^2}{4} dx$$

$$\Rightarrow \int_0^2 y dx = \left[ \frac{x^3}{12} \right]_0^2$$

$$\Rightarrow \int_0^2 y dx = \frac{8}{12} = \frac{2}{3}$$

8. (c) intersecting

**Explanation:** Here  $(a_1, b_1, c_1) = (2, 3, 4)$

and,  $(a_2, b_2, c_2) = (3, 4, 5)$

Consider 
$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

We get

$$\begin{vmatrix} 0 & 0 & 0 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$$

Since the shortest distance is zero hence the lines are intersecting.

9. (d)  $0 \leq \theta \leq \frac{\pi}{2}$

**Explanation:**  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$ ,

Also,  $\vec{a} \cdot \vec{b} \geq 0$

$$\Rightarrow |\vec{a}| |\vec{b}| \cos \theta \Rightarrow \cos \theta \leq 0 \Rightarrow 0 \leq \theta \leq \frac{\pi}{2}$$

10. (a)  $y^2 dx + (x^2 - xy - y^2) dy = 0$

**Explanation:** it is a homogeneous differential equation, because the degree of each individual term is same i.e. 2.

11. (d) 4

**Explanation:** Reqd. area sq. units

$$= \int_0^2 (y-2) dy + \int_2^0 (2-y) dy + \int_0^4 2 dy$$

$$= \left[ \frac{y^2}{2} - 2y \right]_0^2 + \left[ 2y - \frac{y^2}{2} \right]_2^0 + [2y]_0^4$$

$$= (2-4) - (4-2) + 8 = 4$$

12. (c)  $\frac{1}{6a^3} \log \left| \frac{a^3 + x^3}{a^3 - x^3} \right| + C$

**Explanation:**  $I = \int \frac{x^2}{(a^3)^2 - (x^3)^2} dx$

Let  $x^3 = t$

$\Rightarrow 3x^2 dx = dt$

$\Rightarrow x^2 dx = \frac{dt}{3}$

$\Rightarrow I = \frac{1}{3} \int \frac{dt}{(a^3)^2 - t^2}$

We know,  $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$= \frac{1}{6a^3} \log \frac{a^3 + t}{a^3 - t} + c$

put  $t = x^3$

$= \frac{1}{6a^3} \log \frac{a^3 + x^3}{a^3 - x^3} + c$

13. (a)  $a = \frac{1}{2}$

**Explanation:**  $a = \frac{1}{2}$

14. (b)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**Explanation:** In the given question the given matrix is an Identity matrix, and  $I^n = I.I.I$  .....  $I(n \text{ times}) = I$ .

15. (c)  $2^6$

**Explanation:**  $|A| = d$

$|\text{adj } A| = |A|^{n-1}$

Here,  $n = 3$ ,  $|A| = 8$

$|\text{adj } A| = 8^2$

$|\text{adj } A| = (2^3)^2 = 2^6$

16. (c) a unique solution

**Explanation:** a unique solution

The given system of equations can be written in matrix form as follows:

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -18 \end{bmatrix}$$

$$AX = B$$

Here,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 6 \\ -18 \end{bmatrix}$$

$$|A| = 1(-1-2) - 1(-3-6) + 1(3+3)$$

$$= -3 + 3 + 6$$

$$= 6 \text{ not equal to } 0.$$

So, the given system of equations has a unique solution.

$$17. (b) \left[ \frac{-\pi}{2}, \frac{\pi}{2} \right] - \{0\}$$

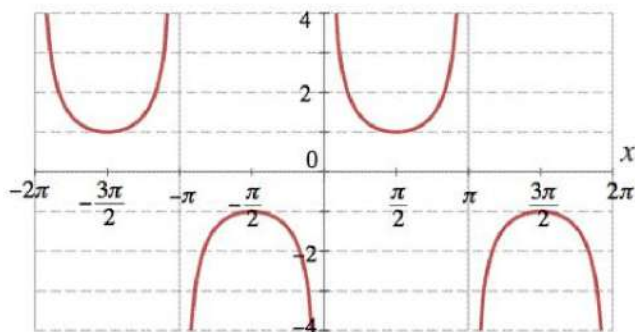
**Explanation:** To Find: The range of  $\operatorname{cosec}^{-1}(x)$

Here, the inverse function is given by  $y = f^{-1}(x)$

The graph of the function  $\operatorname{cosec}^{-1}(x)$  can be obtained from the graph of

$Y = \operatorname{cosec}^{-1}(x)$  by interchanging  $x$  and  $y$  axes. i.e, if  $a, b$  is a point on  $Y = \operatorname{cosec} x$  then  $b, a$  is the point on the function  $y = \operatorname{cosec}^{-1}(x)$

Below is the Graph of the range of  $\operatorname{cosec}^{-1}(x)$



From the graph, it is clear that the range of  $\operatorname{cosec}^{-1}(x)$  is restricted to interval

$$\left[ \frac{-\pi}{2}, \frac{\pi}{2} \right] - \{0\}$$

$$18. (b) \cos \frac{y-1}{x} = a$$

**Explanation:**  $\frac{dy}{dx} = \cos^{-1} a$

$$\int dy = \cos^{-1} a \int dx$$

$$y = x \cos^{-1} a + c$$

When  $y = 1$ ,  $x = 0$ , then  $1 = 0 \cos^{-1} a + c$   $c = 1$

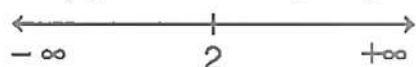
$$\therefore y = x \cos^{-1} a + 1$$

$$\therefore \frac{y-1}{x} = \cos^{-1} a$$

19. (b) Both A and R are true but R is not the correct explanation of A.

**Explanation:** We have,  $f(x) = x^2 - 4x + 6$

$$\text{or } f'(x) = 2x - 4 = 2(x - 2)$$



Therefore,  $f'(x) = 0$  gives  $x = 2$ .

Now, the point  $x = 2$  divides the real line into two disjoint intervals namely,  $(-\infty, 2)$  and  $(2, \infty)$ .

In the interval  $(-\infty, 2)$ ,  $f'(x) = 2x - 4 < 0$ .

Therefore,  $f$  is strictly decreasing in this interval.

Also, in the interval  $(2, \infty)$ ,  $f'(x) > 0$  and so the function  $f$  is strictly increasing in this interval.

Hence, both the statements are true but Reason is not the correct explanation of Assertion.

20. (b) Both A and R are true but R is not the correct explanation of A.

**Explanation:** Both A and R are true but R is not the correct explanation of A.

### Section B

$$21. \text{ Let } \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = y$$

$$\Rightarrow \cos y = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos y = -\cos \frac{\pi}{4}$$

$$\Rightarrow \cos y = \cos\left(\pi - \frac{\pi}{4}\right) = \cos \frac{3\pi}{4}$$

Since, the principal value branch of  $\cos^{-1}$  is  $[0, \pi]$ .

Therefore, Principal value of  $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$  is  $\frac{3\pi}{4}$ .

$$22. \text{ Given that, } x \frac{dy}{dx} = x + y$$

$$\Rightarrow \frac{dy}{dx} = \frac{x+y}{x}$$

This is a homogeneous differential equation

Substituting,  $y = vx$  and  $\frac{dy}{dx} = v + x \frac{dv}{dx}$ , we get

$$v + x \frac{dv}{dx} = \frac{x+vx}{x}$$

$$\Rightarrow v + x \frac{dv}{dx} = 1 + v$$

$$\Rightarrow x \frac{dv}{dx} = 1 + v - v$$

$$\Rightarrow x \frac{dv}{dx} = 1$$

$$\Rightarrow dv = \frac{1}{x} dx$$

Integrating both sides, we get

$$\int dv = \int \frac{1}{x} dx$$

$$\Rightarrow v = \log |x| + C$$

Putting  $v = \frac{y}{x}$ , we get

$$\Rightarrow \frac{y}{x} = \log |x| + C$$

$$\Rightarrow y = x \log |x| + Cx$$

Hence,  $y = x \log |x| + Cx$  is the required solution..

23. Here

$$A = \begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$$

$$\text{Hence, } |A| = \begin{vmatrix} 5-x & x+1 \\ 2 & 4 \end{vmatrix}$$

$$= (5-x) \times 4 - (x+1) \times 2 \dots (\text{Expanding along } R_1)$$

$$\Rightarrow |A| = 18 - 6x$$

For A to be a singular matrix,  $|A|$  has to be 0.

$$\text{Therefore, } 18 - 6x = 0 \Rightarrow x = 3.$$

OR

For the given system, we have

$$D = \begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix} = 2 \times 5 - (-1) \times 3 = 13 \neq 0$$

$$D_1 = \begin{vmatrix} 17 & -1 \\ 6 & 5 \end{vmatrix} = 85 + 6 = 91 \text{ and } D_2 = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix} = 12 - 51 = -39.$$

So, by Cramer's rule, we obtain

$$x = \frac{D_1}{D} = \frac{91}{13} = 7 \text{ and } y = \frac{D_2}{D} = \frac{-39}{13} = -3$$

Hence,  $x = 7$  and  $y = -3$  is the required solution

$$24. \text{ Given, } \vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\therefore \lambda \vec{a} = 2\lambda\hat{i} - 4\lambda\hat{j} + 5\lambda\hat{k}$$

For a unit vector, its magnitude equals to 1.

$$\text{We know that for any vector } \vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \text{ the magnitude } |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\therefore |\lambda \vec{a}| = \sqrt{(2\lambda)^2 + (4\lambda)^2 + (5\lambda)^2} = 1$$

$$\Rightarrow 45\lambda^2 = 1$$

$$\Rightarrow \lambda^2 = \frac{1}{45} = \frac{1}{(3\sqrt{5})^2}$$

$$\Rightarrow \lambda = \pm \frac{1}{3\sqrt{5}}$$

25. Let A, B, C and D denote the events of not getting a white ball in first, second, third and fourth draw respectively. Since the balls are drawn with replacement.

Therefore, A, B, C and D are independent events such that

$$P(A) = P(B) = P(C) = P(D)$$

There are 16 balls, out of which 11 are not white. Therefore,  $P(A) = P(B) = P(C) = P(D)$

$$= \frac{11}{16}$$

Therefore, the required probability is given by,  $P(A \cap B \cap C \cap D) = P(A)P(B)P(C)P(D) =$

$$\left(\frac{11}{16}\right)^4.$$

### Section C

26. According to the question,  $f\left(\frac{3x-4}{3x+4}\right) = x + 2$

$$\text{Let } \frac{3x-4}{3x+4} = t \Rightarrow 3x-4 = 3xt + 4t$$

$$3x - 3xt = 4t + 4$$

$$\Rightarrow x = \frac{4t+4}{3(1-t)}$$

$$\therefore f(t) = \frac{4t+4}{3(1-t)} + 2$$

$$\therefore f(x) = \frac{4x+4}{3(1-x)} + 2$$

$$= \frac{4x+4+4-4}{3(1-x)} + 2$$

$$= \frac{4x-4+8}{3(1-x)} + 2$$

$$= \frac{4(x-1)+8}{3(1-x)} + 2$$

$$= \frac{4(x-1)}{3(1-x)} + \frac{8}{3(1-x)} + 2$$

$$= -\frac{4}{3} - \frac{8}{3(x-1)} + 2$$

$$f(x) = \frac{2}{3} - \frac{8}{3(x-1)}$$

$$\therefore \int f(x)dx = \frac{2}{3}x - \frac{8}{3}\ln |x-1| + c$$

27. The given differential equation is,

$$\cos x \frac{dy}{dx} - \cos 2x = \cos 3x$$

$$\Rightarrow dy = \frac{\cos 3x + \cos 2x}{\cos x} dx$$

$$\Rightarrow dy = \frac{4\cos^3 x - 3\cos x + 2\cos^2 x - 1}{\cos x} dx$$

$$\Rightarrow dy = (4\cos^2 x - 3 + 2\cos x - \sec x) dx$$

$$\Rightarrow dy = [2(2\cos^2 x - 1) - 1 + 2\cos x - \sec x] dx$$

$$\Rightarrow dy = (2\cos 2x - 1 + 2\cos x - \sec x) dx$$

integrating both sides, we get

$$\int dy = \int (2\cos 2x - 1 + 2\cos x - \sec x) dx$$

$$\Rightarrow y = \sin 2x - x + 2\sin x - \log |\sec x + \tan x| + C$$

Hence,  $y = \sin 2x - x + 2\sin x - \log |\sec x + \tan x| + C$  is the solution to the given differential equation.

OR

The given differential equation is,

$$(2x^2 y + y^3) dx + (xy^2 - 3x^3) dy = 0$$

$$\frac{dy}{dx} = - \frac{(2x^2 y + y^3)}{(xy^2 - 3x^3)}$$

It is a homogeneous equation

$$\text{Put } y = vx \text{ and } x \frac{dv}{dx} + v = \frac{dy}{dx}$$

$$\text{So, } x \frac{dv}{dx} + v = \frac{(2x^2 vx + v^3 x^3)}{(-xv^2 x^2 + 3x^3)}$$

$$x \frac{dv}{dx} = \frac{(2v + v^3)}{(-v^2 + 3)} - v$$

$$x \frac{dv}{dx} = \frac{(2v^3 - v)}{(-v^2 + 3)}$$

Integrating Both Sides We get,

$$\int \frac{(-v^2+3)}{(2v^3-v)} dv = \int \frac{1}{x} dx$$

$$\frac{(-v^2+3)}{v(2v^2-1)} = \frac{A}{v} + \frac{Bv+C}{(2v^2-1)}$$

$$3 - v^2 = A(2v^2 - 1) + (Bv + C)v$$

$$3 - v^2 = (2A + B)v^2 + Cv - A$$

Comparing the coefficient of like power of v

$$A = -3$$

$$C = 0$$

$$\text{and } 2A + B = -1$$

$$\Rightarrow B = 5$$

$$\text{So, } \int \frac{-3}{v} dv + \int \frac{5v}{2v^2-1} dv = \int \frac{1}{x} dx$$

$$-3 \int \frac{1}{v} dv + \frac{5}{4} \int \frac{4v}{2v^2-1} dv = \int \frac{1}{x} dx$$

$$-3 \log v + \frac{5}{4} \log (2v^2 - 1) = \log x + \log c$$

$$-12 \log v + 5 \log (2v^2 - 1) = 4 \log x + 4 \log c$$

$$\frac{(2v^2-1)^5}{v^{12}} = x^4 c^4$$

$$x^2 c^4 y^{12} = (2y^2 - x^2)^5$$

28. Given vectors are,  $\vec{a} = \lambda \hat{i} + \hat{j} + 4\hat{k}$ ,  $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

The projection of  $\vec{a}$  along  $\vec{b}$

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$\vec{a} \cdot \vec{b} = (\lambda \hat{i} + \hat{j} + 4\hat{k}) \cdot (2\hat{i} + 6\hat{j} + 3\hat{k})$$

$$= 2\lambda + 6 + 12$$

$$= 2\lambda + 18$$

$$|\vec{b}| = \sqrt{2^2 + 6^2 + 3^2}$$

$$= \sqrt{49} = 7$$

Given, the projection of vector a along vector b is 4.

$$\therefore \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = 4$$

$$\Rightarrow \frac{2\lambda + 18}{7} = 4$$

$$\Rightarrow 2\lambda + 18 = 28$$

$$\Rightarrow 2\lambda = 10$$

$$\Rightarrow \lambda = 5$$

OR

Given position vectors of points A, B & C are:-

$$\vec{A} = (3\hat{i} - 2\hat{j} + 4\hat{k}), \vec{B} = (\hat{i} + \hat{j} + \hat{k}), \vec{C} = (-\hat{i} + 4\hat{j} - 2\hat{k})$$

$\rightarrow$

$$\Rightarrow AB = \text{Position vector of B} - \text{position vector of A}$$

$$= (\hat{i} + \hat{j} + \hat{k}) - (3\hat{i} - 2\hat{j} + 4\hat{k}) = (-2\hat{i} + 3\hat{j} - 3\hat{k})$$

$\rightarrow$

$$\Rightarrow AC = \text{Position vector of C} - \text{position vector of A}$$

$$= (-\hat{i} + 4\hat{j} - 2\hat{k}) - (3\hat{i} - 2\hat{j} + 4\hat{k}) = (-4\hat{i} + 6\hat{j} - 6\hat{k})$$

$$\Rightarrow (\vec{AB} \times \vec{AC}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & -3 \\ -4 & 6 & -6 \end{vmatrix} = \hat{i} \begin{vmatrix} 3 & -3 \\ -4 & 6 \end{vmatrix} - \hat{j} \begin{vmatrix} -2 & 3 \\ -4 & 6 \end{vmatrix} + \hat{k} \begin{vmatrix} -2 & 3 \\ -4 & 6 \end{vmatrix}$$

$$= \hat{i}(-18 + 18) - \hat{j}(-12 + 12) + \hat{k}(-12 + 12) = 0$$

Hence, proved that A, B and C are collinear.

29. Let the given integral be,

$$I = \int (x+1) \sqrt{x^2 - x + 1} dx$$

$$\text{Also, } x+1 = \lambda \frac{d}{dx} (x^2 - x + 1) + \mu$$

$$\Rightarrow x+1 = \lambda(2x-1) + \mu$$

$$\Rightarrow x+1 = (2\lambda)x + \mu - \lambda$$

Equating the coefficient of like terms

$$2\lambda = 1$$

$$\Rightarrow \lambda = \frac{1}{2} \text{ And}$$

$$\Rightarrow \mu - \lambda = 1$$

$$\Rightarrow \mu - \frac{1}{2} = 1$$

$$\Rightarrow \mu = \frac{3}{2}$$

$$\therefore I = \int \left[ \frac{1}{2}(2x-1) + \frac{3}{2} \right] \sqrt{x^2 - x + 1} dx$$

$$= \frac{1}{2} \int (2x-1) \sqrt{x^2 - x + 1} dx + \frac{3}{2} \int \sqrt{x^2 - x + 1} dx$$

$$= \frac{1}{2} \int (2x-1) \sqrt{x^2 - x + 1} dx + \frac{3}{2} \int \sqrt{x^2 - x + \frac{1}{4} - \frac{1}{4} + 1} dx$$

$$\begin{aligned}
&= \frac{1}{2} \int (2x-1) \sqrt{x^2-x+1} dx + \frac{3}{2} \int \sqrt{\left(x-\frac{1}{2}\right)^2 + \frac{3}{4}} dx \\
&= \frac{1}{2} \int (2x-1) \sqrt{x^2-x+1} dx + \frac{3}{2} \int \sqrt{\left(x-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx
\end{aligned}$$

$$\begin{aligned}
&\text{Let } x^2 - x + 1 = t \\
&\Rightarrow (2x-1)dx = dt
\end{aligned}$$

$$\begin{aligned}
\therefore I &= \frac{1}{2} \int \sqrt{t} dt + \frac{3}{2} \left[ \frac{x-\frac{1}{2}}{2} \sqrt{\left(x-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \right. \\
&\quad \left. \frac{3}{8} \log \left| x - \frac{1}{2} + \sqrt{\left(x-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \right| \right] + C \\
&= \frac{1}{3} (x^2 - x + 1)^{\frac{3}{2}} + \frac{3}{8} (2x-1) \sqrt{x^2-x+1} + \frac{9}{16} \log \left| x - \frac{1}{2} + \sqrt{x^2-x+1} \right| + C
\end{aligned}$$

OR

We know that

$$\begin{aligned}
\sin x &= \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \quad \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \\
\therefore \frac{1}{3 + 2 \sin x + \cos x} &= \frac{1}{3 + 2 \left( \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) + \left( \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} \\
&= \frac{\left( 1 + \tan^2 \frac{x}{2} \right)}{3 \left( 1 + \tan^2 \frac{x}{2} \right) + 4 \tan \frac{x}{2} + \left( 1 - \tan^2 \frac{x}{2} \right)}
\end{aligned}$$

$$\begin{aligned}
 &= \frac{\sec^2 \frac{x}{2} dx}{3 + 3\tan^2 \frac{x}{2} + 4\tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} \\
 \therefore \int_0^x \frac{1}{3 + 2\sin x + \cos x} dx &= \int_0^x \frac{\sec^2 \frac{x}{2} dx}{2\tan^2 \frac{x}{2} + 4\tan \frac{x}{2} + 4}
 \end{aligned}$$

Let  $\tan \frac{x}{2} = t$

Differentiating w.r.t.  $x$ , we get

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = \pi \Rightarrow t = \infty$$

$$\begin{aligned}
 \therefore \int_0^x \frac{\sec^2 \frac{x}{2} dx}{2\tan^2 \frac{x}{2} + 4\tan \frac{x}{2} + 4} &= \int_0^\infty \frac{dt}{t^2 + 2t + 2} \\
 &= \int_0^\infty \frac{dt}{(t+1)^2 + 1} \\
 &= \left[ \tan^{-1}(t+1) \right]_0^\infty \\
 &= \tan^{-1}(\infty) - \tan^{-1}(0+1) \\
 &= \tan^{-1}(\infty) - \tan^{-1}(1) \\
 &= \tan^{-1}\left(\tan \frac{\pi}{2}\right) - \tan^{-1}\left(\tan \frac{\pi}{4}\right) \\
 &= \frac{\pi}{2} - \frac{\pi}{4} \\
 &= \frac{2\pi - \pi}{4} \\
 &= \frac{\pi}{4}
 \end{aligned}$$

$$\therefore \int_0^\pi \frac{1}{3 + 2\sin x + \cos x} dx = \frac{\pi}{4}$$

30. ATQ,

$$x = ae^\theta(\sin \theta - \cos \theta), y = ae^\theta(\sin \theta + \cos \theta)$$

$$\Rightarrow \frac{dx}{d\theta} = a \left[ e^{\theta} \frac{d}{d\theta} (\sin\theta - \cos\theta) + (\sin\theta - \cos\theta) \frac{d}{d\theta} (e^{\theta}) \right] \text{ and}$$

$$\frac{dy}{d\theta} = a \left[ e^{\theta} \frac{d}{d\theta} (\sin\theta + \cos\theta) + (\sin\theta + \cos\theta) \frac{d}{d\theta} (e^{\theta}) \right]$$

$$\Rightarrow \frac{dx}{d\theta} = a \left[ e^{\theta} (\cos\theta + \sin\theta) + (\sin\theta - \cos\theta) e^{\theta} \right] \text{ and}$$

$$\frac{dy}{d\theta} = a \left[ e^{\theta} (\cos\theta - \sin\theta) + (\sin\theta + \cos\theta) e^{\theta} \right]$$

$$\Rightarrow \frac{dx}{d\theta} = a \left[ 2e^{\theta} \sin\theta \right] \text{ and } \frac{dy}{d\theta} = a \left[ 2e^{\theta} \cos\theta \right]$$

$$\therefore \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a \left( 2e^{\theta} \cos\theta \right)}{a \left( 2e^{\theta} \sin\theta \right)} = \cot\theta$$

$$\frac{dy}{dx} = \cot\theta$$

The differentiation of the given function y wrt x is as above.

31. The equation of parabola is

$$y = x^2 - 5x + 15 \dots\dots\dots(1)$$

The equation of line is

$$y = 3x + 3 \dots\dots\dots(2)$$

From (1) and (2), we get

$$3x + 3 = x^2 - 5x + 15$$

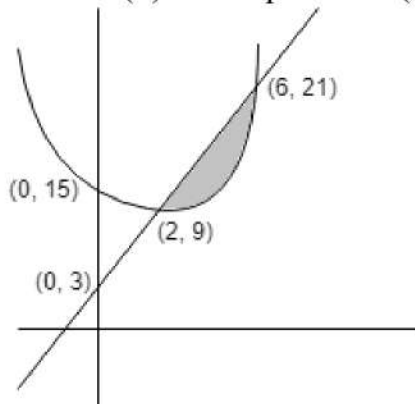
$$x^2 - 8x + 12 = 0$$

$$(x - 2)(x - 6) = 0$$

$$x = 2, 6$$

Therefore, from (1),  $y = 9, 21$

$\therefore$  line (2) meets parabola (1) in (2, 9) and (6, 21).



Required area = the area of the segment of the parabola  $y = x^2 - 5x + 15$  cut off by the straight line  $y = 3x + 3$

$$= \int_2^6 [(3x + 3) - (x^2 - 5x + 15)] dx \quad [\because \text{Area} = \int_a^b (y_2 - y_1) dx]$$

$$= \int_2^6 (-x^2 + 8x - 12) dx$$

$$\begin{aligned}
&= \left[ -\frac{x^3}{3} + 4x^2 - 12x \right]_2^6 \\
&= (-72 + 144 - 72) - \left( -\frac{8}{3} + 16 - 24 \right) \\
&= 0 + \frac{32}{3} \\
&= \frac{32}{3} \text{ sq.units}
\end{aligned}$$

#### Section D

32. Converting the inequations into equations, we obtain the following equations:

$$x + y = 4, 3x + 8y = 24, 10x + 7y = 35, x = 0 \text{ and } y = 0.$$

These equations represent straight lines in XOY-plane.

The line  $x + y = 4$  meets the coordinate axes at  $A_1 (4, 0)$  and  $B_1 (0, 4)$ . Join these points to obtain the line  $x + y = 4$ .

The line  $3x + 8y = 24$  meets the coordinate axes at  $A_2 (8, 0)$  and  $B_2 (0, 3)$ . Join these points to obtain the line  $3x + 8y = 24$ .

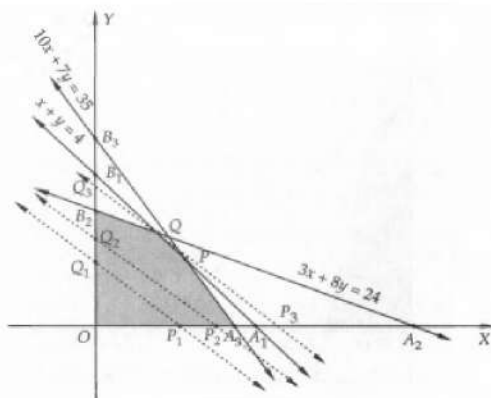
The line  $10x + 7y = 35$  cuts the coordinates axes at  $A_3 (3.5, 0)$  and  $B_3 (0, 5)$ . These points are joined to obtain the line  $10x + 7y = 35$ .

Also,  $x = 0$  is the y-axis and  $y = 0$  is the x-axis.

The feasible region of the LPP is shaded in Figure.

The coordinates of the corner points of the feasible region  $OA_3PQB_2$  are  $O (0, 0)$ ,  $A_3(3.5,$

$0)$ ,  $P(\frac{7}{5}, \frac{5}{3})$ ,  $Q(\frac{8}{5}, \frac{12}{5})$  and  $B_2(0, 3)$ .



Now, we take a constant value, say 10 for  $Z$ . Putting  $Z = 10$  in  $Z = 5x + 7y$ , we obtain the

line  $5x + 7y = 10$ . This line meets the coordinate axes at  $P_1 (2, 0)$  and  $Q(0, \frac{10}{7})$ . Join these

points by a dotted line. Now, move this line parallel to itself in the increasing direction away from the origin.  $P_2Q_2$  and  $P_3Q_3$  are such lines. Out of these lines locate a line farthest from the origin and has at least one common point to the feasible region  $OA_3$

$PQB_2$ . Clearly,  $P_3 Q_3$  is such line and it passes through the vertex  $Q(\frac{8}{5}, \frac{12}{5})$  of the feasible region.

Hence  $x = \frac{8}{5}$  and  $y = \frac{12}{5}$  gives the maximum value of  $Z$ .

The maximum value of  $Z$  is given by

$$Z = 5 \times \frac{8}{5} + 7 \times \frac{12}{5} = 24.8$$

33. Here  $R$  is a relation on  $N \times N$ , defined by  $(a, b) R (c, d) \Leftrightarrow a + d = b + c$  for all  $(a, b), (c, d) \in N \times N$

We shall show that  $R$  satisfies the following properties

i. Reflexivity:

We know that  $a + b = b + a$  for all  $a, b \in N$ .

$\therefore (a, b) R (a, b)$  for all  $(a, b) \in (N \times N)$

So,  $R$  is reflexive.

ii. Symmetry:

Let  $(a, b) R (c, d)$ . Then,

$(a, b) R (c, d) \Rightarrow a + d = b + c$

$\Rightarrow c + b = d + a$

$\Rightarrow (c, d) R (a, b)$ .

$\therefore (a, b) R (c, d) \Rightarrow (c, d) R (a, b)$  for all  $(a, b), (c, d) \in N \times N$

This shows that  $R$  is symmetric.

iii. Transitivity:

Let  $(a, b) R (c, d)$  and  $(c, d) R (e, f)$ . Then,

$(a, b) R (c, d)$  and  $(c, d) R (e, f)$

$\Rightarrow a + d = b + c$  and  $c + f = d + e$

$\Rightarrow a + d + c + f = b + c + d + e$

$\Rightarrow a + f = b + e$

$\Rightarrow (a, b) R (e, f)$ .

Thus,  $(a, b) R (c, d)$  and  $(c, d) R (e, f) \Rightarrow (a, b) R (e, f)$

This shows that  $R$  is transitive.

$\therefore R$  is reflexive, symmetric and transitive

Hence,  $R$  is an equivalence relation on  $N \times N$

OR

Given that,  $\forall a, b \in Z$   $aRb$  if and only  $a-b$  is divisible by  $n$

Now

I. Reflexive

$aRa \Rightarrow (a - a)$  is divisible by  $n$ , which is true for any integer 'a' as '0' is divisible by  $n$ .

Hence,  $R$  is reflexive.

II. Symmetric

$aRb$

$\Rightarrow a-b$  is divisible by  $n$ .

$\Rightarrow -b + a$  is divisible by  $n$ .

$\Rightarrow -(b - a)$  is divisible by  $n$ .

$\Rightarrow (b - a)$  is divisible by  $n$ .

$\Rightarrow bRa$

Hence,  $R$  is symmetric.

III. Transitive

Let  $aRb$  and  $bRc$

$\Rightarrow (a-b)$  is divisible by  $n$  and  $(b-c)$  is divisible by  $n$

$\Rightarrow (a-b) + (b-c)$  is divisible by  $n$

$\Rightarrow$  (a-c) is divisible by n

$\Rightarrow$  aRc

Hence, R is transitive.

So, R is an equivalence relation.

34.  $\vec{a}_1 = \hat{i} + \hat{j}, \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$

$$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$$

$$\begin{aligned}\vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} \\ &= \hat{i}(-2 + 5) - \hat{j}(4 - 3) + \hat{k}(-10 + 3) \\ &= 3\hat{i} - \hat{j} - 7\hat{k}\end{aligned}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{9 + 1 + 49} = \sqrt{59}$$

$$\text{Also, } (\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (3\hat{i} - \hat{j} - 7\hat{k})(\hat{i} - \hat{k}) = 3 + 7 + 0 = 10$$

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \frac{10}{\sqrt{59}}$$

OR

$$\vec{r} = (2i - j + 2k) + \lambda(3i + 4j + 2k)$$

Then, in Cartesian form, we have

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = \lambda \dots (i)$$

coordinates of any point on (i) is ,

$$3\lambda + 2, 4\lambda - 1, 2\lambda + 2$$

The equation of plane is

$$\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

$$x - y + z = 5 \dots (ii)$$

If the point  $(3\lambda + 2, 4\lambda - 1, 2\lambda + 2)$  lies on (ii), then

$$(3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) = 5$$

$$\lambda + 5 = 5$$

$$\lambda = 0$$

we get (2, -1, 2) as the coordinate of the point of intersection of the given line and the plane.

Now distance between the points (-1, -5, -10) and (2, -1, 2)

$$\begin{aligned}&= \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} \\ &= 13\end{aligned}$$

35. Let  $u = \sin^{-1} \left( \frac{2x}{1+x^2} \right)$

Put  $x = \tan \theta$

$$\Rightarrow u = \sin^{-1} \left( \frac{2 \tan \theta}{1 + \tan^2 \theta} \right)$$

$$\Rightarrow u = \sin^{-1}(\sin 2\theta) \dots(i)$$

Let  $v = \cos^{-1} \left( \frac{1-x^2}{1+x^2} \right)$

$$\Rightarrow v = \cos^{-1} \left( \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right)$$

$$\Rightarrow v = \cos^{-1}(\cos 2\theta) \dots(ii)$$

Here,  $0 < x < 1$

$$\Rightarrow 0 < \tan \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

So, from equation (i),

$$u = 2\theta \quad \left[ \text{since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \right]$$

$$\Rightarrow u = 2 \tan^{-1} x \dots [\text{Since, } x = \tan \theta]$$

Differentiating it with respect to  $x$ ,

$$\frac{du}{dx} = \frac{2}{1+x^2} \dots(iii)$$

from equation (ii),

$$v = 2\theta \quad \left[ \text{since, } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right]$$

$$\Rightarrow v = 2 \tan^{-1} x \dots [\text{Since, } x = \tan \theta]$$

Differentiating it with respect to  $x$ ,

$$\frac{dv}{dx} = \frac{2}{1+x^2} \dots(iv)$$

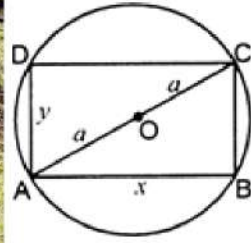
Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2} = 1$$

### Section E

36. Read the text carefully and answer the questions:

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)



- (i) Let 'y' be the breadth and 'x' be the length of rectangle and 'a' is radius of given circle.

$$\text{From fig } 4a^2 = x^2 + y^2$$

$$\Rightarrow y^2 = 4a^2 - x^2$$

$$\Rightarrow y = \sqrt{4a^2 - x^2}$$

$$\text{Perimeter (P)} = 2x + 2y = 2\left(x + \sqrt{4a^2 - x^2}\right)$$

- (ii)

$$\text{We know that } P = 2\left(x + \sqrt{4a^2 - x^2}\right)$$

$$\text{Critical points to maximize perimeter } \frac{dP}{dx} = 0$$

$$\Rightarrow \frac{dp}{dx} = 2\left(1 + \frac{1}{2\sqrt{4a^2 - x^2}}(-2x)\right) = 0$$

$$2\left(\frac{\sqrt{4a^2 - x^2} - x}{\sqrt{4a^2 - x^2}}\right) = 0$$

$$\Rightarrow \sqrt{4a^2 - x^2} = x$$

$$\Rightarrow 4a^2 - x^2 = x^2$$

$$\Rightarrow 2a^2 = x^2$$

$$\Rightarrow x = \pm\sqrt{2a}$$

$$\text{when } x = \sqrt{2a}, y = \sqrt{2a}$$

$$\text{when } x = -\sqrt{2a} \text{ not possible as 'x' is length critical point is } (\sqrt{2a}, \sqrt{2a})$$

(iii)

$$\frac{dp}{dx} = 2 \left( 1 + \frac{1}{2\sqrt{4a^2 - x^2}}(-2x) \right)$$

$$\frac{d^2P}{dx^2} = -2 \left( \frac{\sqrt{4a^2 - x^2} - (x) \left( \frac{-2x}{2\sqrt{4a^2 - x^2}} \right)}{(4a^2 - x^2)} \right)$$

$$= -2 \left( \frac{(4a^2 - x^2) + x^2}{(4a^2 - x^2)^{3/2}} \right)$$

$$\Rightarrow \left. \frac{d^2P}{dx^2} \right]_{x=a\sqrt{2}} = -2 \left( \frac{4a^2}{(4a^2 - 2a^2)^{3/2}} \right) = \frac{-2}{(2\sqrt{2})a} < 0$$

Perimeter is maximum at a critical point.

OR

From the above results know that  $x = y = \sqrt{2}a$

$a$  = radius

Here,  $x = y = 10\sqrt{2}$

Perimeter =  $P = 4 \times \text{side} = 40\sqrt{2}$  cm

**37. Read the text carefully and answer the questions:**

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats, and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold by school A, B, C are given below.



| Article | School | A  | B  | C  |
|---------|--------|----|----|----|
| Fans    |        | 40 | 25 | 35 |
| Mats    |        | 50 | 40 | 50 |
| Plates  |        | 20 | 30 | 40 |

(i) *Fans Mats Plates*

$$P = \begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix}$$

$$Q = \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} Fans \\ Mats \\ Plates \end{matrix}$$

(ii) Clearly, total funds collected by each school is given by the matrix

$$PQ = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$$

$$= \begin{bmatrix} 1000 + 5000 + 1000 \\ 625 + 4000 + 1500 \\ 875 + 5000 + 2000 \end{bmatrix} = \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}$$

$\therefore$  Funds collected by school A is ₹7000.

Funds collected by school B is ₹6125.

Funds collected by school C is ₹7875.

(iii)

$$\text{New price matrix } Q = 20\% \times \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} Fans \\ Mats \\ Plates \end{matrix}$$

$$\Rightarrow Q = \begin{bmatrix} 25 + 25 \times 0.20 \\ 100 + 100 \times 0.20 \\ 50 + 50 \times 0.20 \end{bmatrix} \begin{matrix} Fans \\ Mats \\ Plates \end{matrix}$$

$$Q = \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix} \begin{matrix} Fans \\ Mats \\ Plates \end{matrix}$$

$$\text{New price matrix } Q = 20\% \times \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} Fans \\ Mats \\ Plates \end{matrix}$$

$$\Rightarrow Q = \begin{bmatrix} 25 + 25 \times 0.20 \\ 100 + 100 \times 0.20 \\ 50 + 50 \times 0.20 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

$$Q = \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

OR

$$PQ = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix}$$

$$PQ = \begin{bmatrix} 1200 + 6000 + 1200 \\ 750 + 4800 + 1800 \\ 1050 + 6000 + 2400 \end{bmatrix} = \begin{bmatrix} 8400 \\ 7350 \\ 9450 \end{bmatrix}$$

Total fund collected = 8400 + 7350 + 9450 = ₹25,200

**38. Read the text carefully and answer the questions:**

In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03.



- (i) Let A be the event of committing an error and  $E_1$ ,  $E_2$  and  $E_3$  be the events that Govind, Priyanka and Tahseen processed the form.

$$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$$

$$P\left(\frac{A}{E_1}\right) = 0.06, P\left(\frac{A}{E_2}\right) = 0.04, P\left(\frac{A}{E_3}\right) = 0.03$$

Using Bayes' theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P\left(\frac{A}{E_1}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) \cdot P\left(\frac{A}{E_3}\right)}$$

$$= \frac{0.5 \times 0.06}{0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03} = \frac{30}{47}$$

$$\therefore \text{Required probability} = P\left(\frac{\bar{E}_1}{A}\right)$$

$$= 1 - P\left(\frac{E_1}{A}\right) = 1 - \frac{30}{47} = \frac{17}{47}$$

(ii) Let A be the event of committing an error and  $E_1$ ,  $E_2$  and  $E_3$  be the events that Govind, Priyanka and Tahseen processed the form.

$$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$$

$$P\left(\frac{A}{E_1}\right) = 0.06, P\left(\frac{A}{E_2}\right) = 0.04, P\left(\frac{A}{E_3}\right) = 0.03$$

$$P(A \cap E_2) = P\left(\frac{A}{E_2}\right) \cdot P(E_2)$$

$$\Rightarrow 0.04 \times 0.2 = 0.008$$